This chapter presents you with material that is probably a step up in difficulty. However, later chapters, as well as Part 2 of the FRM will assume knowledge of the material presented herein. This chapter covers important material: the calculation of volatility from historical data, the Black–Scholes–Merton differential equation, risk-neutral valuation, the Black–Scholes–Merton option pricing formulas, implied volatilities, and the impact of dividends. Make sure that these are terms you are comfortable with.

Stock *prices* are distributed according to the *lognormal* distribution. It thus follows that the stock *returns* are *normally* distributed. The expected rate of return in a short period of time is equal to . However, the expected continuously compounded rate of return over a longer period of time is equal to

The price of European options can be derived by either solving the Black-Scholes-Merton partial differential equation, or by using risk-neutral valuation. However, note that it is important that you understand that we are not assuming risk neutrality. It just happens that the value of a derivative security is independent of risk preferences.

Volatility is the only parameter in the Black–Scholes pricing model that cannot be directly observed. If we are given a market price for an option, we solve for the volatility that makes the equation work. Implied volatility is the volatility the produces a model price equal to the observed market price. The solution requires an iterative search procedure since we cannot solve directly for the volatility.

The Black–Scholes is quite flexible and robust. Indeed it can be extended to the case of options on an underlying stock paying dividends, by discounting the stock price by the value of the dividends. Furthermore, the B-S model can be used to value a warrant however, some adjustments need to be made; perhaps the most important [to understand] is the “hair-cut”.

For American options on a dividend paying stock Fisher Black suggested an approximate procedure for taking account of early exercise in call options. This involves calculating the prices of a European option that mature at times T and t(n), where the expiration times represent the expiration of the American option, and expiration immediately prior to the ex-dividend date, respectively. The (American) option price is then set equal to the greater of the two. Black’s approximation is used extensively in industry, and has proved surprisingly accurate. Therefore, traders often use it, rather than more computationally intensive models when time of execution is important.